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SIMPLE PUSH-OVER ANALYSIS OF ASYMMETRIC BUILDINGS

VOJKO KILAR AND PETER FAJFAR

Faculty of Civil and Geodetic Engineering, Institute of Structural and Earthquake Engineering, University of Ljubljana, Jamova 2, 1000 Ljubljana, Slovenia

SUMMARY

A simple method for the non-linear static analysis of complex building structures subjected to monotonically increasing horizontal loading (push-over analysis) is presented. The method is designed to be a part of new methodologies for the seismic design and evaluation of structures. It is based on the extension of a pseudo-three-dimensional mathematical model of a building structure into the non-linear range. The structure consists of planar macroelements. For each planar macroelement, a simple bilinear or multilinear base shear–top displacement relationship is assumed. By a step-by-step analysis an approximate relationship between the global base shear and top displacement is computed. During the analysis the development of plastic hinges throughout the building can be monitored. The method has been implemented into a prototype computer program. In the paper the mathematical model, the base shear–top displacement relationships for different types of macroelements, and the step-by-step computational procedure are described. The method has been applied for the analysis of a symmetric and an asymmetric variant of a seven-storey reinforced concrete frame–wall building, as well as for the analysis of a complex asymmetric 21-storey reinforced concrete wall building. The influence of torsion on structural behaviour is discussed.

KEY WORDS: seismic analysis; non-linear analysis; push-over analysis; building structures; asymmetric structures; computer program

INTRODUCTION

Recently it has been widely recognized that changes are needed in the existing seismic design methodology implemented in codes based on the assumption of linear elastic structural behaviour (e.g. References 1 and 2). Complex analyses, such as non-linear time history analysis of Multi-Degree-of-Freedom (MDOF) mathematical models, are not practical for everyday design use and are not appropriate as a code requirement. The development of a rational methodology that is applicable to the seismic design of new structures as well as to the seismic evaluation and strengthening of existing structures, which takes full advantage of presently available ground motion information and engineering knowledge, and yet is flexible enough to permit the incorporation of new knowledge as it becomes available, has become the focus of several major efforts throughout the world (e.g. SEAOC Vision 2000, ATC-33, ATC-34 and BSSC NEHRP updates in the U.S.A., Eurocode 8 in Europe, and PRESSS design guidelines for RC buildings in Japan[3]). In the majority of cases, non-linear static analysis under monotonically increasing lateral loading (push-over analysis) is an important part of the new methodology. It represents a relatively simple option to estimate non-linear structural performance. In somewhat different formats, push-over analysis has been proposed, formulated and evaluated in several research studies (e.g. References 4–11). In all of the studies, with the exception of the work by Moghadam and Tso,[10] only symmetric, i.e. planar systems were investigated.

Push-over analysis can be performed using the well-known programs for static and dynamic non-linear analysis, e.g. DRAIN-2DX[12] or IDARC,[13] that are restricted to planar structures, or DRAIN-3DX,[12] DRAIN-BUILDINGS[12] or CANNY[14] which permit a three-dimensional analysis. However, in the case of a large and complex asymmetric building structure, the use of a general computer program is time-consuming and impractical (mainly in the definition of the model and in the interpretation and checking of the results) even if the analysis is restricted to statics. Therefore, some attempts have been made to study the

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basic features of the inelastic seismic response of asymmetric building structures and to simplify the analysis procedure (e.g. References 15–19). A list of publications issued before the end of 1994 has been compiled by Rutenberg and co-authors.[20]

At the University of Ljubljana a method for simplified push-over analysis of asymmetric building structures has been developed, which is intended to achieve a satisfactory balance between required reliability and applicability for everyday design use, and which could contribute to the practical implementation of new trends in seismic design.[21] It is based on the extension of a pseudo-three-dimensional mathematical model of a building structure into the inelastic range. The method was implemented in a prototype of the interactive and user-friendly computer program NEAVEK. In the paper the method is briefly described and applied to the analysis of two test examples.

PSEUDO-THREE-DIMENSIONAL MATHEMATICAL MODEL

A pseudo-three-dimensional mathematical model consists of assemblages of two-dimensional macroelements (substructures) such as frames, walls, coupled walls and walls on columns that may be oriented arbitrarily in plane. Each macroelement is assumed to resist load only in its own plane, but the building as a whole can resist load in any direction. The macroelements are connected at each floor level by diaphragms that are assumed to be rigid in their own planes and have no out-of-plane flexural stiffness. The model has three degrees of freedom for each floor level (two horizontal translations and one rotation about the vertical axis). All other degrees of freedom are eliminated by static condensation on the macroelement level, by assuming rigid links, or ignored. Masses are lumped at the floor levels. The compatibility of axial deformations in columns common to more than one frame or in intersecting shear walls is neglected. It is considered that for most buildings this is an acceptable approximation, with the possible exception of some tall slender buildings or tube-type structures. If the frames do not intersect at right angles, coupling of the frames through column bending is ignored.

The advantages of a pseudo-three-dimensional model over a fully three-dimensional model are easier data preparation, easier interpretation of the results, and higher computational efficiency. In spite of its obvious limitations,[22] the pseudo-three-dimensional model has been implemented, in different forms, in computer codes for linear analysis (e.g. the TABS family of programs[23]) and widely used by the engineering profession in design and teaching worldwide. In Slovenia, the pseudo-three-dimensional model was implemented in the EAVEK program.[24,25] The program, which was developed in its original version in the early seventies, has become standard analysis software in Slovenian design offices. In the EAVEK program, the condensed flexibility matrices for 'standard' macroelements (i.e. walls, coupled walls, walls on columns and regular orthogonal frames) are determined by closed-form analytical formulae, or, in the case of coupled walls, by static analysis at the macroelement level. The condensed flexibility matrices for 'non-standard' macroelements (e.g. irregular frames) can be computed by any program for the analysis of plane frames and transferred to the EAVEK program as input data. The corresponding condensed stiffness matrix of a macroelement is obtained by inversion of the condensed flexibility matrix. This matrix is then transformed from the local (element) co-ordinate system to the global (structure) co-ordinate system. The structural stiffness matrix is determined in terms of two translations and one rotation for each floor by summing the transformed matrices of all macroelements. In Figure 1 the 'standard' macroelements used in the EAVEK program and their mathematical models are presented. The expressions for the coefficients of the condensed flexibility matrices are described in detail in References 24 and 26, and are summarized in the appendix.

Only one attempt to extend the pseudo-three-dimensional model into the non-linear range is known to the authors. In 1977, the program DRAIN-TABS[27] for the non-linear dynamic analysis of building structures was developed. This program combines the features of TABS and DRAIN-2D.[28] Non-linear analysis is performed for each two-dimensional frame as in the DRAIN-2D program. At each step the condensed stiffness matrices associated with horizontal floor displacements are calculated for each frame, and then combined to form the stiffness matrix for the complete structure. Unlike the two 'parent' programs (TABS and DRAIN-2D), DRAIN-TABS has been only rarely used.

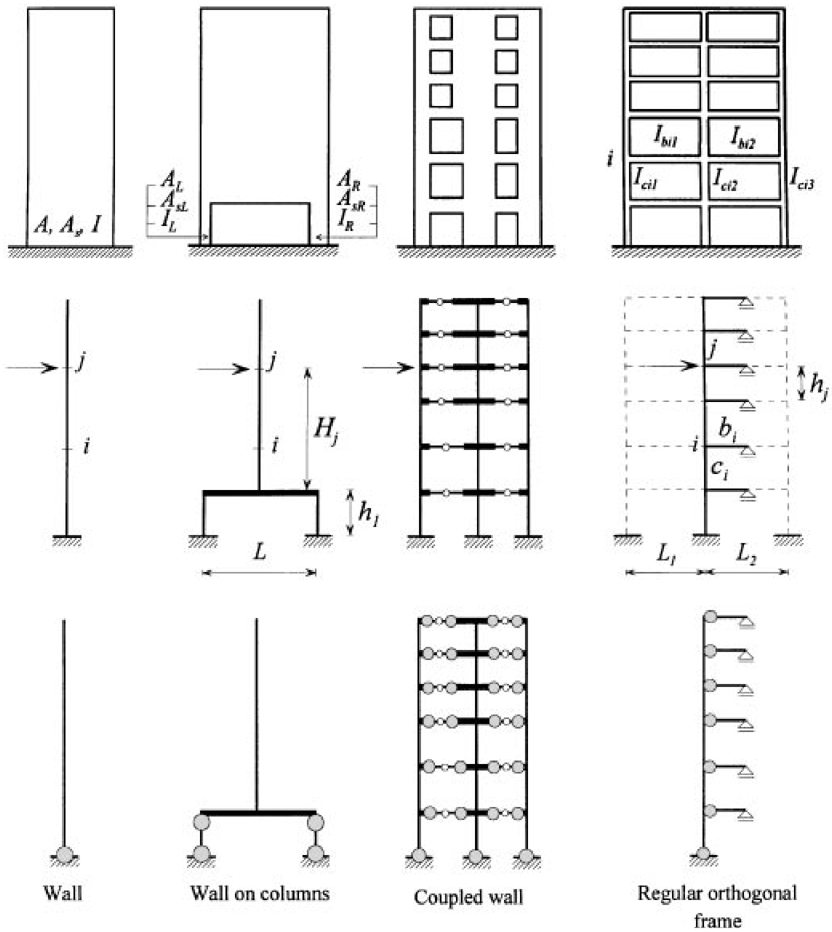

Figure 1. 'Standard' macroelements, mathematical models for elastic analysis, and plastic mechanisms. (For the frame only the global mechanism is shown. Other mechanisms are shown in Figure 2)

A much simpler extension of the pseudo-three-dimensional model into the non-linear range has been used in the proposed method, which is restricted to static analysis, and implemented in the NEAVEK program (Non-linear EAVEK). In the DRAIN-TABS program the non-linear analysis of individual macroelements is performed in detail with the DRAIN-2D program. In the program NEAVEK, however, for each macroelement a bilinear or multilinear base shear–top displacement relationship is determined based on the initial stiffness, the strength at which the assumed plastic mechanism forms, and assumed post-yield stiffness. Thus, using the proposed method a stepwise elastic analysis of the structure can be performed. In each step, the stiffnesses of at least one macroelement and of the whole structure change.

NON-LINEAR BASE SHEAR–TOP DISPLACEMENT RELATIONSHIPS FOR MACROELEMENTS

In this section approximate base shear–top displacement relationships are developed for four 'standard' macroelements (Figure 1). It is planned that additional macroelements will be added during the further development of the procedure.

For each macroelement, one or more possible plastic mechanisms are assumed. For three macroelement types (walls, walls on columns and frames) elastic behaviour is assumed until the plastic mechanism is formed. The elastic stiffness can be based on uncracked, cracked or some average section properties. The base

shear V is defined as the sum of horizontal forces F_i over all the storeys. The top displacement D can be determined as

$$D = \sum_{i=1}^{n} F_i d_{ni} \tag{1}$$

where F_i is the horizontal force in the ith storey, and d_{ni} are the coefficients of the condensed flexibility matrix that represent the top displacement (in the nth storey) due to unit horizontal force in the ith storey (see the appendix). After the formation of the plastic mechanism, the force–displacement relationship is governed by the post-yield stiffness, which is arbitrarily assumed on the macroelement level. So, the base shear–top displacement relationship of a macroelement is bilinear, provided that the vertical distribution of lateral loading is constant. If this distribution changes during the loading history (this always happens, in principle, when one of the elements in the structural system yields) then the slope of the base shear–top displacement line also changes.

In the case of a coupled wall, static analysis of the mathematical model shown in Figure 1 is needed in order to determine the elastic base shear–top displacement relationship. Gradual formation of the plastic mechanism is assumed. Consequently, the base shear–top displacement relation is piecewise linear. The stiffness changes after the yielding of different elements of the coupled wall (beams, walls).

A step-by-step strategy is used for the solution of this problem, as discussed in the next section. At every step, all the results (the internal forces and displacements) obtained by applying an increment of the external loading to the current mathematical model, are superposed on the results that correspond to the end of the previous step.

In the following subsections the yield mechanisms and post-yield behaviour for each of the 'standard' macroelements will be discussed. The formulae for the determination of the condensed flexibility matrices for the three macroelement types (walls, walls on columns and frames) are given in the appendix.

Walls

Walls are treated as cantilever beam elements. Only one plastic mechanism, i.e. the formation of a plastic hinge at the bottom of the wall (Figure 1), is assumed. A plastic hinge at the base appears when the base bending moment M becomes equal to the yield moment M_y. After the plastic mechanism has been formed, a flexural spring with a small stiffness is introduced in the mathematical model at the base. The wall remains part of the structural model of the complete structure. Any increase in the base shear of the wall due to an increase in the external loading depends on the assumed post-yield stiffness, and is typically small. However, the elastic upper part of the wall may substantially influence the distribution of external loading onto the other macroelements. The gradual spreading of yielding along the height of wall is not taken into account in the present version of the computational procedure. However, the results indicate if the spreading of yielding has occurred.

Coupled walls

For coupled walls with *one row of beams* the mechanism similar to that shown in Figure 1 (hinges at the base of all walls and in all the beams) is assumed. The mechanism is formed in three steps. The behaviour of the macroelement is elastic until all the beams yield simultaneously in the entire row, at both ends. The yielding condition is defined as

$$\sum_{i=1}^{n} 2M_i = \sum_{i=1}^{n} 2M_{yi} \tag{2}$$

where M_i is the bending moment at the end of the beam in storey i and M_{yi} is the yield moment of the same cross-section. Both values are assumed to be equal at both ends of the beam. The summation is performed over all the beams in a row (n is the number of storeys). It is assumed that all hinges develop simultaneously in the entire row of beams at both of their ends. The bending moments M_i, as well as the other internal forces

and lateral displacements, are determined by elastic static analysis of the mathematical model shown in Figure 1. For subsequent load increments the coupled wall is divided into two separated walls, which are treated as two separate macroelements. Zero post-yield stiffness is assumed for beams. The plastic mechanism for the coupled wall occurs when both walls yield at the base (typically not at the same time).

A similar procedure can be applied to a *coupled wall with several rows of beams*. After the first row of beams yields, such a wall is divided into two separate parts. At least one of them is still a coupled wall. The procedure is repeated under increasing lateral loading until the coupled wall disintegrates to isolated walls, and until each of these walls yields at the base.

The yield moment at the base of a wall depends on the axial force. In a wall, which is a part of a coupled wall, the axial force changes while the horizontal loading increases. After yielding of the beams at both sides of the wall, the axial force remains constant.

Several approximations are included in the procedure described above. The assumption of all beams in one row yielding at the same time seems to be acceptable for most practical applications, as shown in Reference 21. More questionable may be the assumption regarding the sequence of the formation of plastic hinges (i.e. the plastic hinge at the base of the wall should not occur before the adjacent beams yield) and the assumption that the beams yield in flexure (i.e. shear failure of beams, associated with an overall strength deterioration, should not occur). It is planned that, in the future, the computational procedure will be complemented in order to be able to deal with the majority of possible cases. For the time being, however, a warning has been incorporated into the computer program, indicating a deviation from the assumed model. Deviations will, in most cases, be associated with cases where the principles of correct earthquake resistant design have been violated.

Walls on columns

Structural walls often have a large opening in the first storey. Unfortunately, such walls are often used for architectural reasons in spite of possible concentration of damage and low energy dissipation capacity. They can be modelled as cantilever beams supported on two columns connected together by a stiff horizontal beam. It is assumed that the behaviour of the macroelement is elastic until plastic hinges appear simultaneously at the top and bottom of both columns, and a plastic mechanism is formed (Figure 1). At this moment the sum of the bending moments in all four critical sections is equal to the sum of the yield moments in the same cross-sections. The post-yield behaviour of the macroelement is modelled by reducing the flexural stiffness of columns to a small percentage of the initial stiffness. The internal forces and the coefficients of the condensed flexibility matrix (see the appendix) can be determined by the methods of structural mechanics.

In the supporting columns, typically large axial forces due to horizontal loading occur. These forces should be taken into account when determining the yield moments.

Frames

Only regular frames that form an orthogonal grid of beams and columns can be processed at the present time. For such frames the coefficients of the condensed flexibility matrix can be approximately determined by closed-form equations (see the appendix). In all cases it is assumed that the yield moments of all beams in a storey are equal. The same assumption is used for all columns in a storey. If this is not the case, average yield moments are used.

It is assumed that the behaviour of the frame is elastic until a plastic mechanism is formed. Three main types of plastic mechanism, as proposed by Mazzolani and Piluso,[29] are anticipated (Figure 2). The global-type mechanism is a special case of the type 2 mechanism. The multipliers for the horizontal forces for the different types of mechanism (defined as the ratio between the forces at the formation of the plastic mechanism and the applied forces F_i) are given in Reference 29 and elsewhere. The multipliers represent event factors (discussed in the next section) for the first step of analysis. All possible mechanisms are checked ($3n - 1$ values). The expected plastic mechanism is controlled by the smallest multiplier.

After the formation of a plastic mechanism, the stiffness of all the beams and all the columns decrease to a small percentage of their initial values. In fact, in all cases (with the exception of the global mechanism) part

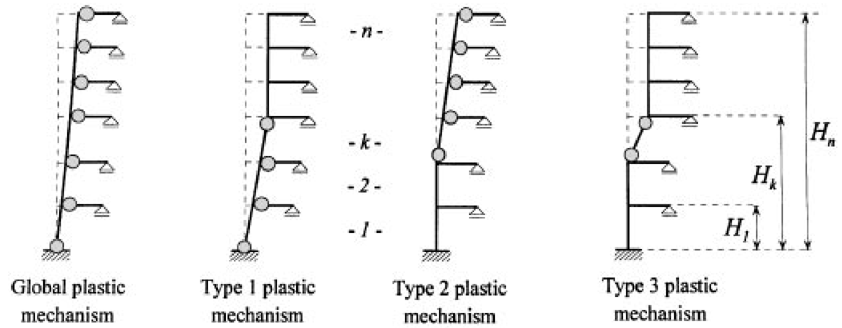

Figure 2. Anticipated plastic mechanisms for the frame

of the frame remains elastic after the formation of the plastic mechanism. This feature is neglected in the present version of the computational procedure.

The described procedure yields a fairly accurate estimate for the strength of the frame, assuming that its elements do not fail due to secondary effects (e.g. buckling) before the plastic mechanism occurs. The assumed bilinear force–displacement relationship corresponds to the simultaneous yielding where all plastic hinges occur in the same moment. In a more frequent case of sequential yielding, the estimates of displacements corresponding to the base shears near the formation of a plastic mechanism are typically not accurate. Nevertheless, in most cases the simple bilinear force–displacement relation represents an acceptable approximation having in mind all the uncertainties involved in seismic design.

METHOD OF ANALYSIS

Analysis is performed as a sequence of linear analyses, using an event-to-event strategy. An event is defined as a discrete change of the structural stiffness due to the formation of a plastic hinge (or the simultaneous formation of several plastic hinges) in a macroelement. Due to piecewise linear force–displacement relationships and an 'exact' determination of all events, the event-to-event procedure does not produce any unbalanced forces.

The computational procedure is as follows:

(1) All the structural data have to be known. In addition to the data needed for elastic analysis, the yield moments for potential plastic hinges are required. If the axial force in the critical cross-section is substantially influenced by the magnitude of the horizontal loading, the effect of changing axial force on the yield moment may be taken into account. The seismic capacity of structural members (including shear strength) is not needed for the analysis. It is needed, however, for the evaluation of the analysis results, for example, for checking if a brittle failure mode occurs before the predicted mechanism can be formed.

(2) The distribution of horizontal static loads over the height of the building is chosen (e.g. an inverted triangular distribution) and the increment of the load magnitude is arbitrarily selected. For asymmetric structures the co-ordinates of the points of load application (usually in the centre of the masses) are specified, as well as the direction of loading.

(3) For the selected load increment, the usual elastic analysis is performed. The global displacement increments, as well as the increments in load distribution, displacements and internal forces are computed for each macroelement.

(4) In the previous section, non-linear base shear–top displacement relationships were determined for several types of macroelements. Using these relationships, event factors α for all anticipated events for all the macroelements are calculated. The event factor is defined as the ratio between the load

increment that causes an event and the selected load increment. Thus, the event factor is a scale factor (multiplier) for the selected external load increment. The smallest event factor defines the event that happens next, and the actual load increment that should be added to the total external loading in order for the next event to occur.

(5) All the response quantities determined as described in Step (3) are scaled with the minimum event factor and added to the results of the previous step. In this way, the solution advances to the next predicted event.
(6) The mathematical model and/or stiffness of the macroelement that triggered the event is changed according to the rules described in the previous section.
(7) Using the new mathematical model, the procedure described in Steps (3)–(6) is repeated.
(8) There are several options for the termination of analysis, e.g. the formation of a plastic mechanism for the whole structure, the exceeding of a prescribed maximum allowable top displacement for a macro-element, or the detection of a brittle failure mode.

The proposed method is illustrated on a simple example of a symmetric building that consists of a wall (element A) and a frame (element B). First, a loading increment is applied and elastic analysis is performed. The distribution of the external load to both elements is determined using the assumption of equal displacements of both elements in each story (the rigid slab assumption). Based on the vertical distribution of horizontal forces, the event factor for the wall and the event factors for all anticipated frame mechanisms are computed. The minimum event factor corresponds to the wall, indicating that the wall yields first. In the next step a new mathematical model for the wall is used. An additional external load increment is applied, and the new distribution of the external forces to the wall and frame is determined by elastic analysis of the new model. Using the new vertical distribution of the horizontal forces, all the assumed frame mechanisms are checked once again. The smallest event factor corresponds to the first storey mechanism (note that in the previous step, under a different load distribution, the global frame mechanism was critical). After yielding of the frame, a plastic mechanism for the whole building is formed. The analysis may be terminated or continued by taking into account the non-zero post-yielding stiffness of both macroelements. The event to event procedure is illustrated in Figure 3.

An equivalent procedure can be used for asymmetric structures. In this case increments in the load and displacement are calculated using a 3D elastic analysis. The described event-to-event procedure remains unchanged. In some cases unloading of a macroelement can occur. The unloading stiffness is equal to the loading stiffness in the same step.

RESULTS OF ANALYSIS

The proposed method is one of the simplest possible analytical tools for determining the main characteristics of non-linear structural behaviour under monotonic static loading. It is based on several simplifying assumptions, and does not pretend to be very accurate. Nevertheless, it can provide fair estimates of several parameters which cannot be predicted by elastic analysis and which represent a basis for the evaluation of structural behaviour during strong earthquakes.

The results of the analysis include, *inter alia*, force–displacement and force–torsional rotation relationships for the whole structure, force–displacement relationships for all macroelements, and the plastic mechanism of the structure. From these results the stiffness and the strength of the structure can be determined and, based on the known target top displacement, global ductility demand, as well as local displacement and ductility demands at the macroelement level, can be estimated. Several directions of horizontal loading and several vertical distributions of horizontal loading can be easily checked. Based on the information obtained by the proposed push-over analysis, the critical parts of the structure can be identified, and several practical issues concerning the design and the retrofit of building structures can be addressed, without performing a complex non-linear dynamic analysis which, in practice, precludes experimentation with alternative solutions, especially if the structural configuration of the system changes between these alternatives. The method is able, at least approximately, to provide answers to many of the relevant questions defined by De la Llera and

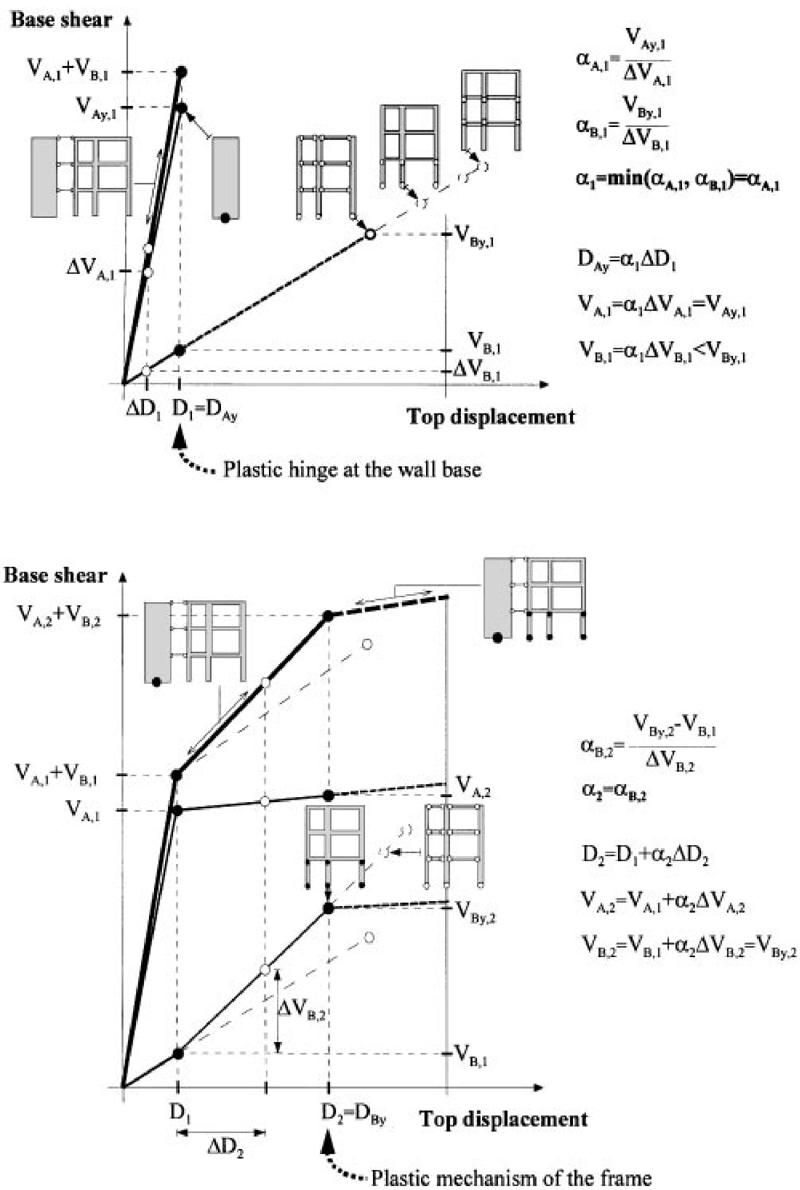

Figure 3. Illustration of the proposed push-over analysis

Chopra[19]: How can we adjust the planwise distribution of stiffness and strength in the system in order to achieve a good performance? How can we localize or spread the damage among the macroelements? What macroelements should be stiffened or stregthened? What is the effect of the orthogonal component of ground motion on the design of macroelements in the direction of the first component of the ground motion? How is the system going to collapse?

Using the global results on the macroelement level, estimates of the local behaviour can be obtained. In the case of structural walls, the deformations imposed at the base of the wall can be determined directly from the known lateral displacement patterns, as demonstrated by Wallace and Moehle.[30] In frames, rough estimates of the behaviour of individual beams and columns can be obtained from the computed displacements and

from the relevant plastic mechanism. For more detailed analysis, post-processing with a program for the non-linear analysis of frame structures is needed.

Push-over analysis has already been widely accepted and incorporated into the seismic design methodologies which are restricted to planar (symmetric) structures (e.g. the N2 method[5,11]). Research is under way aimed at extending the design procedure to asymmetric structures by incorporating 3D push-over analysis. Several problems still have to be solved. One of them is how to combine the loading in two orthogonal directions. Another problem is how to take into account the possibility of the additional ductility demand of elements at the stiff and/or strong edge due to dynamic torsional effects, which can occur in the case of torsionally flexible structures.

COMPUTER PROGRAM

The method was implemented in a prototype of the computer program NEAVEK (Non-linear analysis of multistorey buildings). The program runs in a MS-WINDOWS environment, and provides an interactive, user-friendly and graphically supported environment for the non-linear static analysis of building structures. At each step the program NEAVEK automatically calls the program EAVEK, that performs a linear elastic analysis. In addition to the data needed for the elastic analysis, only the cross-section yield moments (or the axial force–bending moment diagrams, if the influence of the axial force on the yield moment is taken into account) of potential plastic hinges are required. The same mathematical model is used for elastic and for non-linear analysis. This makes the proposed method very practical and easy to use in everyday practice.

EXAMPLES

The proposed procedure can, in principle, be applied to building structures of any material. Our applications so far, however, have been restricted to reinforced concrete buildings. Generally, ductile behaviour of critical cross-sections was assumed. Yield moments were determined by the usual procedures. In this paper two examples will be presented. In the first example the proposed method has been applied to the analysis of the symmetric and an asymmetric variant of a seven-storey building structure. In the second example, a complex (real) 21-storey reinforced concrete building was analysed. The structural system that consists of a large number of relatively thin structural walls, is typical for many regions in the world, including Central and Eastern Europe and South America. Such structural systems behaved well during the 1985 Chile earthquake.

Seven-storey buildings

In the first example two seven-storeyed RC building structures have been analysed (Figure 4). The first building, a symmetric one, completely corresponds to the RC frame–wall building tested in Tsukuba within the framework of the joint U.S.–Japan research project. More data on this structure are given elsewhere.[31] In the second, asymmetric building, the structural wall has been moved from the middle frame II into frame III (in one variant of the asymmetric building the frames in the transverse direction have been omitted). The initial stiffness of all the structural elements is based on their gross cross-sections, an average measured modulus of elasticity of $E = 2{\cdot}5 \times 10^7$ kN/m^2, and a shear modulus of $G = 1{\cdot}0 \times 10^7$ kN/m^2. The following yield moments (in kN m) have been taken into account (in all storeys): columns on corners 310, other columns 384, the wall 14 250, beams (flange in tension) 87, and beams (flange in compression) 230. The frame with the wall is subdivided into two macroelements (one wall and one frame consisting of two identical one-bay frames) by assuming zero rotation and zero vertical displacement at the end of the beams originally connected to the wall. Thus, the mathematical model consists of four macroelements in the longitudinal (X) direction (wall, two three-bay frames and one double one-bay frame) and four identical frames in the transverse (Y) direction (with the exception of one variant of asymmetric building without frames in Y-direction). The flexibility of the spring at the base of the wall simulating post-yield behaviour is determined according to equation (6) in the appendix. A relatively large value of the post-yield stiffness ($p = 0{\cdot}1$) is chosen in order to simulate the observed 3D effects (the outriggering action of frames B and C on the wall) in the

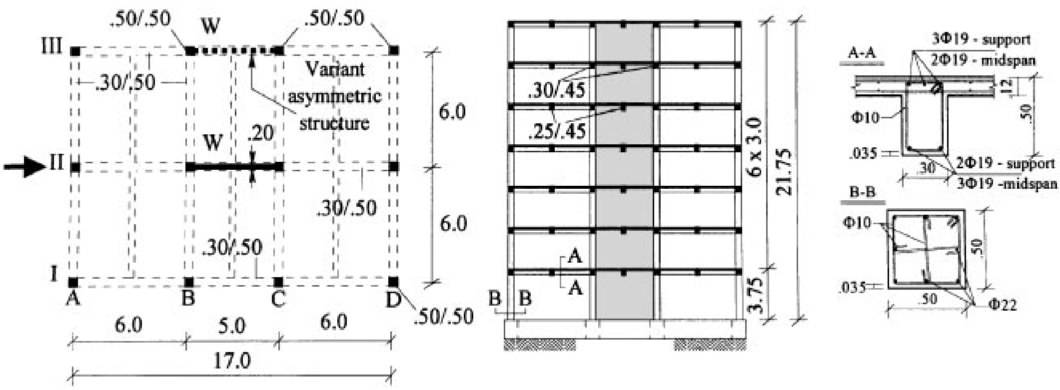

Figure 4. Plan, elevation and typical cross-sections of structural members of the seven-storey buildings

tests. The post-yield stiffnesses of the frame elements are assumed to be equal to 1 per cent of their initial stiffnesses. The horizontal loading is applied in the X-direction in the middle of the building. In one case a simultaneous loading in X- and Y-direction is applied. An inverted triangular distribution of the loading throughout the height of the building is used.

The base shear–top displacement relationships obtained are shown in Figure 5. The top displacements in the X-direction are monitored at the middle frame. The results, obtained by the CANNY program[14] and the envelope of the pseudo-dynamic tests of the full-scale symmetric structure in Tsukuba,[31] are shown for comparison. It should be noted that the behaviour of the structure is largely controlled by the behaviour of the structural wall, especially in the case of the symmetric structure. Consequently, how the structural wall is modelled in a computer program has an important influence on the computed behaviour. The intention of the analysis with the CANNY program was to obtain, independently, a rough estimate of the non-linear behaviour of the asymmetric structure. A simple model for the structral wall was used: the wall, together with the edge columns, was modelled as a panel element. The properties of the whole I-shaped cross-section were attributed to the panel, and a very low stiffness was attributed to the boundary columns. For the other columns the multi-spring model that takes into account bending and shear deformations in two directions, as well as axial deformations, was used. This model takes into account axial compatibility in columns that belong to two frames.

The comparison of base shear–top displacement relationships (Figure 5) demonstrates that the proposed method is able to predict the non-linear force–displacement relation with reasonable accuracy (considering all uncertainties involved in seismic design). If the stiffness is reduced to 72 per cent of the initial value (from the stiffness based on gross cross-sections to an effective stiffness adapted to the test results) very good correlation with the envelope of the pseudo-dynamic tests can be observed (Figure 5(a)). The discussion that follows is based on the results obtained by assuming initial stiffnesses based on gross cross-sections.

In the symmetric structure (Figures 5(a) and 6(a)), the wall (W) yields first, then global plastic mechanisms are formed in the frames in lines I (FI) and III (FIII), and finally in the frame in line II (FII). At this moment, at a base shear of $V = 2510$ kN and a top displacement of $D = 4{\cdot}0$ cm, plastic mechanisms are formed in all the macroelements in the X-direction. In the case of the asymmetric structure with transverse frames, loaded in the X-direction, plastic mechanisms are formed in the following order (Figures 5(b) and 6(b)): frame I (type 1 mechanism with plastic hinges in the columns of the fifth storey), the wall, frame II (global type), and the frame in line III (global type). Plastic mechanisms in all the macroelements in the X-direction are formed at $V = 2460$ kN and $D = 6{\cdot}8$ cm. At this moment, the top displacements in frames I and III amount to 9·7 and 3·8 cm, respectively. An additional increase of the loading in the X-direction causes plastic mechanisms in the frames in lines A (FA) and D (FD), and finally in the frames in lines B (FB) and C (FC).

A comparison of the results for both structures indicates that larger displacements and larger ductilities are required in the case of the asymmetric structure in order to develop the same strength as in the symmetric

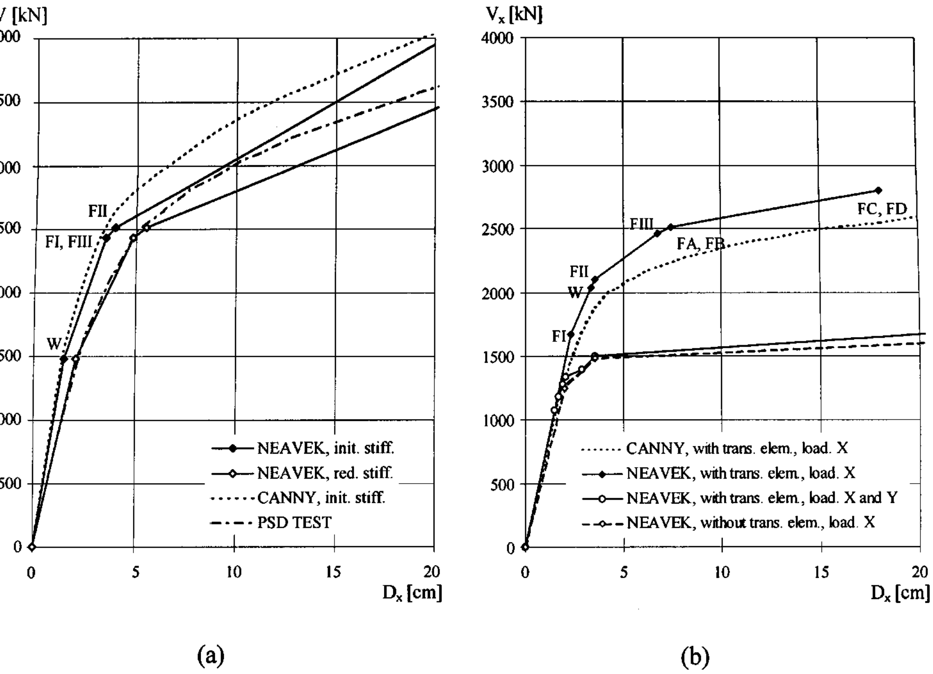

Figure 5. Base shear–top displacement relationships for the seven-storey buildings: (a) symmetric building; (b) asymmetric building (initial stiffness). V_x is the X-component of the base shear and D_x is the X-component of the top displacement of the middle frame. The formation of plastic mechanisms in different macroelements is indicated

structure, especially on the weak side of the building. In other words, at the same displacement the strength of the asymmetric structure is less than the strength of the symmetric one. Furthermore, due to the different vertical distribution of lateral loading on the macroelements of the two structures (Figure 7), the type of plastic mechanism for frame I changes from the global type to type 1. More details about the computed behaviour of different macroelements of both structures can be seen in Figure 6, where base shear–top displacement relationships are shown, and in Figure 7, where the vertical distributions of horizontal loading along the height of the macroelements are presented.

If the asymmetric structure is loaded simultaneously in the X- and Y-direction, the strength in the X-direction is drastically reduced (Figure 5(b)). Due to the yielding of all frames in the weak Y-direction, as well as of two frames (FI and FII) in the X-direction, a torsional plastic mechanism is formed which prevents further increase of lateral loading in the Y-direction. Consequently, the strength of the main structural element, i.e. the wall, and the adjacent frame III, cannot be fully exploited.

Very similar results are obtained if the transverse macroelements, i.e. frames *A–D*, are not taken into account, and the loading is applied only in the X-direction (Figure 5(b)).

21-storey building

In order to demonstrate the capability of the proposed method for the analysis of complex buildings commonly encountered in practice, a 21-storey reinforced concrete structural wall building has been analysed (Figure 8). The building was designed by a structural engineering bureau in Ljubljana according to the seismic code of the former Yugoslavia (Reference 32), using modal analysis. Some simplifications have been made in the test structure regarding the original design. The structure represents the state-of-the-practice in Slovenia. However, it does not necessarily comply with all requirements in modern codes for seismic design. For example, the local stability of the relatively thin walls without boundary elements and without connections to the transverse walls may be questionable.

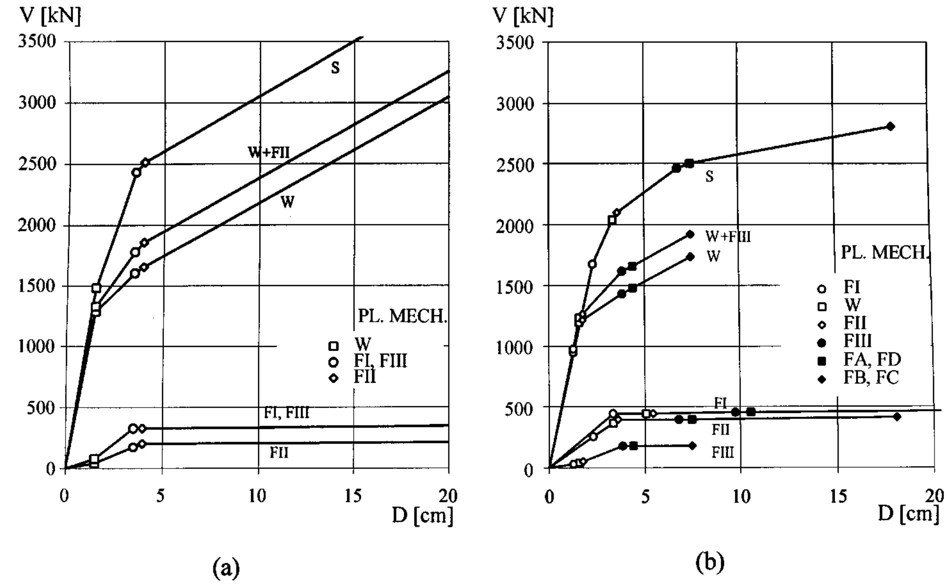

Figure 6. Base shear–top displacement relationships for macroelements (wall W and frames FI, FII and FIII) and for the structure (S): (a) symmetric building; (b) asymmetric building with transverse elements, loading in X-direction. The formation of plastic mechanisms in different macroelements is indicated

The main structural characteristics of the building are as follows. The height of the first, second and all other storeys are 5·0, 3·6 and 3·0 m, respectively. The thickness of all the walls is 20 cm, and the height of all the coupling beams is 70 cm. The mass is the first storey amounts to 1376 tons and in all other stories to 1278 tons. The nominal cylindrical strength of the concrete in the first eight storeys is 41·6 MPa and in all upper storeys 25·0 MPa. All walls are reinforced by two layers of distributed reinforcement (tensional strength 500 MPa), and by additional longitudinal reinforcement at both ends (tensional strength 400 MPa). The reinforcement ratio of the distributed reinforcement is 0·31 per cent in both the longitudinal and the transverse direction. The amount of additional longitudinal reinforcement at each end of the walls depends on the bending moment determined in the analysis, and varies from 0·15 per cent (minimum reinforcement) to about 1.00 per cent of the total cross-section. The longitudinal reinforcement ratio in the coupling beams depends on the bending moment calculated in the analysis, and varies from 0·76 per cent (minimum reinforcement) to about 3·0 per cent. The stiffness and strength of all the elements were determined taking into account rectangular cross-sections, i.e. the influence of flanges in T and I sections was neglected. Initial stiffness was based on gross cross-sections, and the modulus of elasticity was assumed to be $E = 3 \times 10^7$ kN/m^2. Using these data, the following three fundamental periods were calculated: $T_1 = 3{\cdot}07$ s (predominantly the X-direction), $T_2 = 1{\cdot}61$ s (predominantly torsion), and $T_3 = 1{\cdot}40$ s (predominantly the Y-direction). The design base shear values for MSK intensity VIII and low quality of soil amounted to 3·0 and 3·2 per cent of the total mass in the X- and Y-direction, respectively. The load factor amounted to 1·3. When determining the yield moments in the coupled walls, a constant axial force equal to the axial force due to the dead load was assumed. The post-yield stiffness of the walls was set to 1·5 per cent of the initial stiffness of the wall (H_{eq} in equation (6) was taken to be 2/3 of total building height).

The lateral loading was applied at the mass centre separately in the X- and Y-direction. An inverted triangular distribution of the loading along the height of the building was assumed. The relationships between the base shear and top displacement (at the mass centre), as well as between the base shear and top torsion rotation, are shown in Figure 9. The yielding of the different elements is indicated.

From Figure 9 the following observations can be made. Both the stiffness and strength in the Y-direction are much larger than in the X-direction. The overstrength, defined as the maximum strength divided by the

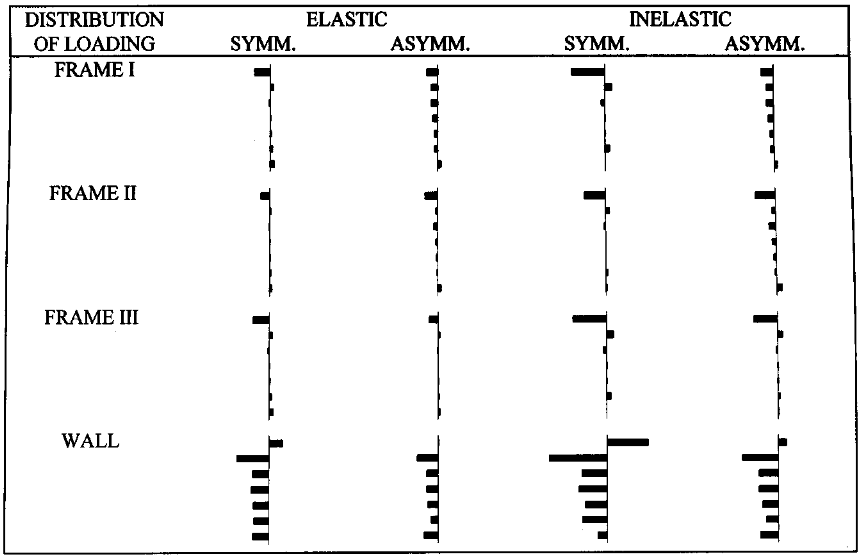

Figure 7. Distribution of horizontal loading throughout the height of the macroelements of the symmetric and asymmetric building with transverse elements in the elastic range (just before the first plastic mechanism is formed) and in the inelastic range (just before plastic mechanisms are formed in all the macroelements in X-direction)

design base shear multiplied by the load factor 1·3, amounts to about 1·4 and 2·3 in the X- and Y-direction, respectively. The computed overstrength is due to the redundancy of the coupled walls, and due to the minimum reinforcement in some of the walls, which is larger than is statically required. Probable additional overstrength due to possible higher material quality and additional overstrength due to the contribution of flanges in the T and I cross-sections have not been included. In the X-direction, the stiffest wall $X8$ yields first. The beams in the coupled wall $X15$ and wall $X2$ then follow. The last macroelement that yields is the coupled wall $X11$. If 1 per cent of the building height (65 cm) is assumed as the target displacement (in a real design the displacement demand has to be determined by analysis), the maximum required ductility of a macroelement amounts to about 3·4 ($X8$). In the case of loading in the Y-direction, the coupled beams in the coupled walls $Y11$, $Y15$, and $Y2$ yield first, followed by walls $Y6$ and $Y2$A. The walls in the coupled wall $Y15$ yield last. The required ductilities, corresponding to a top displacement of 65 cm at the mass centre, are from 11 ($Y11$a) to 7 ($Y15$a) for beams, and from 7 ($Y6$) to 2·5 ($Y15$C) for walls. The torsional rotations are relatively small, and generally increase the displacements of the macroelements on the flexible sides of the elastic structure (the right-hand and bottom sides). However, in the case of the loading in the Y-direction, the direction of the torsional rotation is reversed after yielding of the stiffest wall $Y2$A. The direction is reversed again after the formation of the plastic mechanism in the Y-direction. This example shows that, in the case of non-linear behaviour, it is difficult to define the ‘flexible’ side because it can change during loading history.

The wall model presently employed allows yielding only at the bottom of the wall. In high rise buildings, yielding may spread throughout the height of walls. In the analysed building, the yield moments are slightly exceeded (by up to 23 per cent) in the second, third and fourth storey of walls $X8$, $Y6$ and $Y2$A, indicating yielding. A comparative analysis using a more accurate model allowing vertical spreading of yielding in the walls has produced very similar results.

It is interesting to mention that preliminary calculations of the structure with the same geometry (i.e. stiffness), but with a different strength distribution, produced much larger torsional rotations. This fact suggests that the inelastic torsional response of the building may be very sensitive to the strength-to-stiffness ratio of individual macroelements.

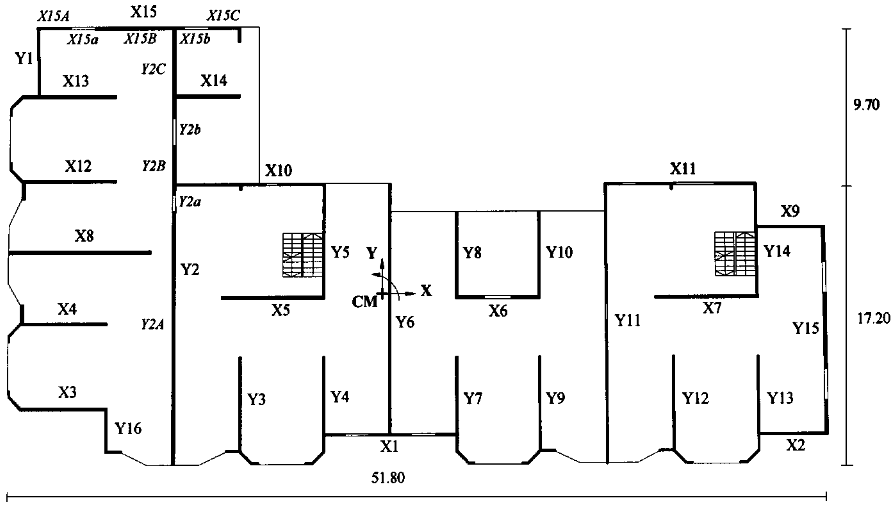

Figure 8. Plan of the 21-storey reinforced concrete building. Within a coupled wall, walls are denoted by *A*, *B* and *C*, and beams are denoted by *a* and *b*

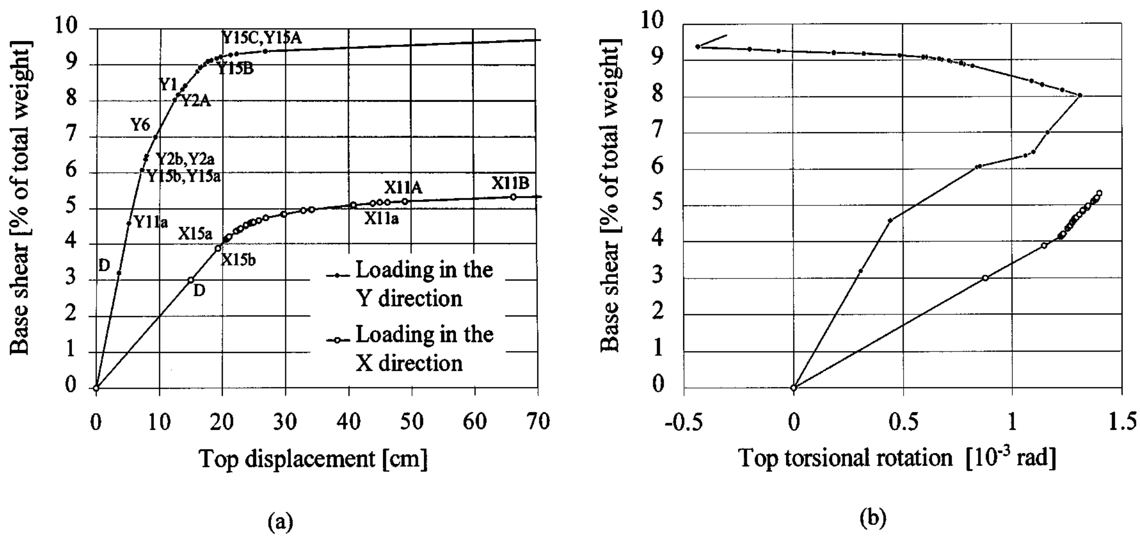

Figure 9. Relationships for the 21-storey building: (a) base shear–top displacement in the direction of loading at the mass centre; (b) base shear–top torsional rotation. Design base shear is indicated by *D*

CONCLUSIONS

The proposed simple procedure for the push-over analysis of building structures is capable of estimating several important characteristics of non-linear structural behaviour, especially the real strength and global plastic mechanism. It also provides data about the sequence of yielding of different parts of the structure, and an estimate of the required ductilities of the different macroelements in relation to the target maximum displacement. The effort needed for data preparation, computation and interpretation of the results is much smaller than in the case of more accurate and sophisticated non-linear analysis methods. Due to its ability to answer several issues concerning the design and retrofit, the proposed procedure could be appropriate for the practical analysis and design of earthquake-resistant building structures and for the evaluation of existing structures.

Illustrative examples, especially the seven-storey building, have clearly demonstrated the unfavourable influence of torsion in asymmetric structures. The results indicate that, in general, larger displacements and larger ductilities are required in an asymmetric structure in order to develop the same strength as in the symmetric structure, especially at the flexible and/or weak side of the building. If a torsional plastic mechanism is formed the available strength of some macroelements cannot be fully exploited. Torsional rotations and the formation of a torsional mechanism strongly depend on the structural elements which resist loads in the direction perpendicular to the direction of the applied loading.

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APPENDIX

Condensed flexibility matrices of 'standard' macroelements

In this appendix, the closed-form expressions for the condensed flexibility matrices of three 'standard' macroelements (a wall, a wall on columns and a frame) are summarized. More details can be found in References 24–26. The condensed flexibility matrix for a coupled wall is determined in the program EAVEK by the usual elastic static analysis of an equivalent frame model.

The coefficient of the condensed flexibility matrix d_{ij} represents the horizontal displacement in storey i due to a unit force in storey j. All the expressions for d_{ij} apply if H_j is greater than or equal to H_i, where H_i and H_j represent the elevations of storeys i and j, respectively. The flexibility matrix is symmetric. The following notation is used: I for moments of inertia of cross-sectional areas (second moment of area), A for cross-sectional areas and A_s for shear areas. E and G are the modulus of elasticity and the shear modulus, respectively. Other quantities are defined in Figure 1.

Walls

The expressions for walls are based on the cantilever beam model. In the case of a constant cross section along the height, d_{ij} can be determined as

$$d_{ij} = \frac{H_i^2}{6EI}(3H_j - H_i) + \frac{H_i}{GA_s} \tag{3}$$

and, in the case of a stepwise constant cross-section as

$$\begin{aligned} d_{ij} &= d_{i-1,j} + d'_{i-1,j}h_i + \frac{1}{6EI_i}(3h_i^2 H_j - H_i^3 - 2H_{i-1}^3 + 3H_i H_{i-1}^2) + \frac{h_i}{G}\left(\frac{1}{A_{si}} - \frac{1}{A_{si-1}}\right) \\ d'_{ij} &= d'_{i-1,j} + \frac{1}{2EI_i}(2h_i H_j - H_i^2 + H_{i-1}^2) + \frac{1}{G}\left(\frac{1}{A_{si}} - \frac{1}{A_{si-1}}\right) \\ d_{0j} &= d'_{0j} = 0 \end{aligned} \tag{4}$$

The coefficient d_{ij}^P of the condensed flexibility matrix after yielding can be determined as

$$d_{ij}^P = d_{ij} + d_\varphi H_i H_j \tag{5}$$

where d_{ij} is defined according to equation (3) or equation (4) and d_φ is the flexibility of the rotational spring that is introduced into the mathematical model after yielding (the rotation due to a unit moment).

d_φ can be approximately related to the assumed post-yield stiffness of the wall, expressed as a fraction of the initial stiffness (pEI), by the equation (Reference 21),

$$d_\varphi \cong \frac{H_{eq}}{3pEI} \tag{6}$$

where H_{eq} is the equivalent height of the wall defined as $H_{eq} = M/V$, M and V being the bending moment and base shear at the base, respectively. H_{eq} depends on the vertical distribution of the lateral loads. In our calculations the values of M and V corresponding to the loading at the formation of the plastic hinge were used.

Walls on two columns

The flexibility matrix for a wall (modelled as a beam) on two columns takes into account, in addition to flexural and shear deformations, the axial deformations of the columns

$$
\begin{aligned}
d_{ij} &= d_u + \frac{d_L d_R}{d_L + d_R} + \frac{h_1}{4L^2 E}\left(\frac{1}{A_L} + \frac{1}{A_R}\right)(2H_j + h_1)(2H_i + h_1) \\
d_L &= \frac{h_1^3}{12EI_L} + \frac{h_1}{GA_{sL}}, \qquad d_R = \frac{h_1^3}{12EI_R} + \frac{h_1}{GA_{sR}}
\end{aligned}
\tag{7}
$$

d_u represents the displacement of the upper part (the wall). It is determined according to equation (3) or equation (4), by measuring the elevation from the first storey, and not from the bottom of the element.

After the formation of the plastic mechanism, small values (depending on the assumed post-yield stiffness) are used for the cross-sectional moments of inertia I_L and I_R in the first storey.

Frames

Simple formulae for d_{ij} for frames can be derived using the following approximations:

(a) the frame is modeled as shown in Figure 1,
(b) the inflection point of each column is at the mid-height, except for the columns in the first floor, where the position of the inflection point is the same as for the corresponding single-storey frame,
(c) axial and shear deformations are neglected and
(d) the stiffness of the part of frame above the point of application of the load is neglected:

$$
\begin{aligned}
d_{11} &= \frac{h_1^2}{12}\left(\frac{1}{c_1} + \frac{1}{4b_1 + 0.33c_1}\right) \\
d_{12} &= d_{1j} = d_{11} + \frac{h_1 h_2}{48b_1 + 4c_1} \\
d_{22} &= d_{11} + \frac{h_2^2}{12}\left(\frac{1}{c_2} + \frac{1}{4b_1} + \frac{1}{4b_2}\right) + \frac{h_1 h_2}{24b_1 + 2c_1} \\
d_{ii} &= d_{i-1,i-1} + \frac{h_i^2}{12}\left(\frac{1}{c_i} + \frac{1}{4b_{i-1}} + \frac{1}{4b_i}\right) + \frac{h_{i-1} h_i}{24b_{i-1}} \\
d_{ij} &= d_{ii} + \frac{h_i h_{i+1}}{48b_i} \\
c_i &= \sum_k \frac{EI_{cik}}{h_i}, \qquad b_i = \sum_k \frac{EI_{bik}}{L_k}
\end{aligned}
\tag{8}
$$

The summation is performed over all the columns and beams in the storey i. Experience has shown that, in spite of the several approximations involved, equations (8) yield quite accurate results, provided that the frame is regular (consisting of an orthogonal grid of beams and columns) and the stiffness of the beams b_i in any storey is not much smaller than the stiffness of the columns c_i in the same storey (e.g. $b_i > 0{\cdot}25c_i$).

After the formation of the plastic mechanism, small values (depending on the assumed post-yield stiffness) are used for the cross-sectional moments of inertia of all the beams (I_b) and all the columns (I_c).

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